Estimate of the number of land transactions along the ByPass from 2005 to 2021, using the Gauss-Newton algorithm.

RAMBININTSOA Franco Patrick, RAKOTO DAVID Séraphin, RAMBININTSOA Tahina, NDRIANAMBININA Djohary, HENIPANALA Mampionona
Université d'Antananarivo - Ecole Supérieure Polytechnique d'Antananarivo– Mention Urbanism Architecture and Civil Engineering – BP 1500 Antananarivo 101 Madagascar -francrambi@yahoo.fr

Abstract
Land along the new roads built on Antananarivo is currently the subject of land speculation. This scientific publication estimates the number of land transactions along the By-Pass from 2005 to 2021. To do this, we use the non-linear mathematical model which uses the Gauss-Newton algorithm.
The result of this modelling leads us to plead for comments in terms of public policies, in particular for the simplification and acceleration of land transfer procedures in a controlled legal framework.

Keywords : Nonlinear Mathematical Model, Land Security, Antananarivo, By-Pass

1- INTRODUCTION

Madagascar has considerable socio-economic and environmental potential, but so far it is in a situation of poor land management. Land along the new roads is subject to land speculation. In this scientific publication, we estimate the number of land transactions along the By-Pass from 2005 to 2021. This modelling will be used as a calibration of the prediction of the "life curve" of the land market along the tracks of the new roads built in Antananarivo, whose urbanization plans are being developed.

2- LITERATURE REVIEW

- Gilli, M. and M. Garbely (1996). Matching, Covers, and Jacobian Matrices. Journal of Economic Dynamics and Control 20, 1541–1556.
- Golub, G. H. and C. F. Van Loan (1989). Matrix Computations. Johns Hopkins. Baltimore.
- Lawson, C. L. and R. J. Hanson (1974). Solving Least Squares Problems. PrenticeHall. Englewood Cliffs, NJ.
- Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling (1986). Numerical Recipes. Cambridge University Press. Cambridge.

3- METHODS

3.1- Numerical resolution of non-linear models by the Gauss-Newton Algorithm

<u>3.1.1- Resolution algorithm</u>

This resolution uses very complex iterative numerical procedures. The idea is to arrive at new coefficients that allow to obtain a linear form, by proceeding to a limited development of Taylor on the basis of initial coefficients or iterative values. Then, apply the OLS to the linear form thus obtained to estimate the new coefficients. Iterations stop more and more as the coefficients become stable. However, "estimator convergence" is a criterion that sanctions the efficiency of the method used (initial values should be closer to optimal values).

Let the non-linear model: $Y_t = (X, a) + e_t$ (1)

With : Y_t = dependent variable (in matrix form)

X_t = vector of explanatory variables (matrix of observations of explanatory variables of dimension « n, k + 1 »)

a = parameter vector to be estimated (dimension « k + 1 »)

e_t = random term or error.

The least squares estimator for the non-linear model (1) is "a" which minimizes the « (a) = $e'e$ = [Y_t − (X, a)]' [Y_t − $f(X, a)$] » as follows (we have k+1 conditions of the 1st order: $\partial S/\partial a$ = 0) :

$$\frac{\partial S}{\partial a} = -2\,\frac{\partial f(X,a)}{\partial a}\,[Y_t - f(X,a)] = 0$$

With : $\frac{\partial f(X,a)}{\partial a}$ = Z(a) = $\begin{pmatrix} \frac{\partial f(X_1,a)}{\partial a_0} & \cdots & \frac{\partial f(X_1,a)}{\partial a_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial f(X_n,a)}{\partial a_0} & \cdots & \frac{\partial f(X_n,a)}{\partial a_k} \end{pmatrix}$ the jacobian matrix J(x). [2]

Note: ($a*$) = matrix calculated for the particular values of a (a = $a*$). Due to Taylor's limited development in the vicinity of « $a*$», we obtain the following approximation to the «t-th observation»:

$f(x_t, a) \simeq f(x_t, a) + \{\frac{\partial f(X_t,a)}{\partial a_0}\,|_{a = a^*} \ldots \ldots \frac{\partial f(X_t,a)}{\partial a_k}\}$ (a- a*)

$\{\frac{\partial f(X_t,a)}{\partial a_0}|_{a = a^*} \ldots \frac{\partial f(X_t,a)}{\partial a_k}\}$ is the matrix of the first derivatives noted ∇f(x) or ∇r(x)'r(x)

Otherwise, in matrix notation:

$f(X, a) \simeq$ f(X,a*) + Z(a*)(a-a*)… … (2)

(2) may also be written:y = f(X,a*) + Z(a*)(a-a*) + e

Ou y = f(X,a*) + Z(a*)a - Z(a*)a* + e … … (3)

By posing: $\bar{y}$(a*) = y - f(X,a*) + Z(a*)a* , then (3) is reduced to:

$\bar{y}$(a*) = Z(a*)a + e …. …. (4)

The estimator of this "linear model (4) " (by OLS) is:

$a^2 = [Z(a^*)'\,Z(a^*)]^{-1}\,Z(a^*)'\,\bar{y}(a^*) = a^* + [Z(a^*)'\,Z(a^*)]^{-1}\,Z(a^*)'[y-f(X,a)]$

We will have « k+1 » new values for the vector « a = a Convergence is achieved when « $\hat{a}$ = $a^p \simeq a^{p-1}$ » (stability of coefficients in the p-th iteration). [3]

The Matlab or Eviews, Rats, etc. can be used to apply non-linear estimation methods.

It should also be noted that the efficiency of non-linear estimation methods is a function of the initial values compatible with the model specification and the data.

Currently, for intrinsically non-linear models or complex non-linear models (difficult to linearize), and there are no estimators as such, optimization algorithms are used to obtain linear forms.

3.1.2- A simple example to understand the Gauss-Newton algorithm

The purpose of this example is to understand the generated algorithm. The data is rather brief and has no relation to the land survey. We will see later its application on the Land with fairly complex real data: the calculations are very long and we use the Matlab for the application on the Land.

Here is the simple example: either the non-linear model **y =** $\boldsymbol{a_1 e^{a_2 x}}$ with the following data:

x_i	0	1	2	3
y_i	2	0,7	0,3	0,1

Note that this model is a generalization of the hyperbolic model with $a_2 < 0$.

Let us also remember that we estimate the coefficients a_1 and a_2.

The residue vector is written: $r(a) = \begin{pmatrix} y_1 - a_1 e^{a_2 x_1} \\ y_2 - a_1 e^{a_2 x_2} \\ y_3 - a_1 e^{a_2 x_3} \\ y_4 - a_1 e^{a_2 x_4} \end{pmatrix}$

The Jacobian matrix is: $J(a) = \begin{pmatrix} \frac{\partial r_1}{\partial a_1} & \frac{\partial r_1}{\partial a_2} \\ \frac{\partial r_2}{\partial a_1} & \frac{\partial r_2}{\partial a_2} \\ \frac{\partial r_3}{\partial a_1} & \frac{\partial r_3}{\partial a_2} \\ \frac{\partial r_4}{\partial a_1} & \frac{\partial r_4}{\partial a_2} \end{pmatrix} = \begin{pmatrix} -e^{a_2 x_1} & -a_1 x_1 e^{a_2 x_1} \\ -e^{a_2 x_2} & -a_1 x_2 e^{a_2 x_2} \\ -e^{a_2 x_3} & -a_1 x_3 e^{a_2 x_3} \\ -e^{a_2 x_4} & -a_1 x_4 e^{a_2 x_4} \end{pmatrix}$

The primary derived matrix is: $[J(a^{(k)})]^t \, . \, r(a) = \begin{pmatrix} \frac{\partial y}{\partial a_1} \\ \frac{\partial y}{\partial a_2} \end{pmatrix} = \begin{pmatrix} -\sum_{i=1}^{4} r_i e^{a_2 x_i} \\ -\sum_{i=1}^{4} r_i a_1 x_i e^{a_2 x_i} \end{pmatrix}$

The continuation of the calculation is quite long and requires programming. A repetition of the following idea (for standard languages) gives the algorithm:

```
Choose x^(0)
for k = 0, 1, 2,…. until convergence do
    calculer J(x^(k))
    solve  [J(x^(k))]^t . J(x^(k)) . a^(k)_GN = - [J(x^(k))]^t . r(x)
    update  a^(k+1) = a^(k) + a^(k)_GN
end for
```

Notes on this example of the Gauss-Newton algorithm

- $a^{(k)}_{GN}$ is the solution to the problem. It is the vector of the parameters to be estimated.
- The programming algorithm consists of a loop and the calculation is done by iterations (k can go up to 10 or 20 or more). This algorithm contains the main lines of the programming. We will see the details of this programming in the land application.
- We do not calculate with programming here. The purpose of this example is to detail the calculation of the residue matrix, the Jacobian matrix and the primary derived matrix. The calculations of these matrices will be integrated into the programming algorithm during the application on the land. [3]

4 FINDINGS: Estimation of the number of land transactions along the ByPass, by "indirect estimation" (with changes over time)

We begin by estimating the number of land transactions over time along the By-Pass. Then we can, later, the relationship between land prices and the number of land transactions.

4.1- Estimated number of land transactions along the By-Pass

Recall the dissemination model translating the models of «life curve of the land market» (Figure 1)

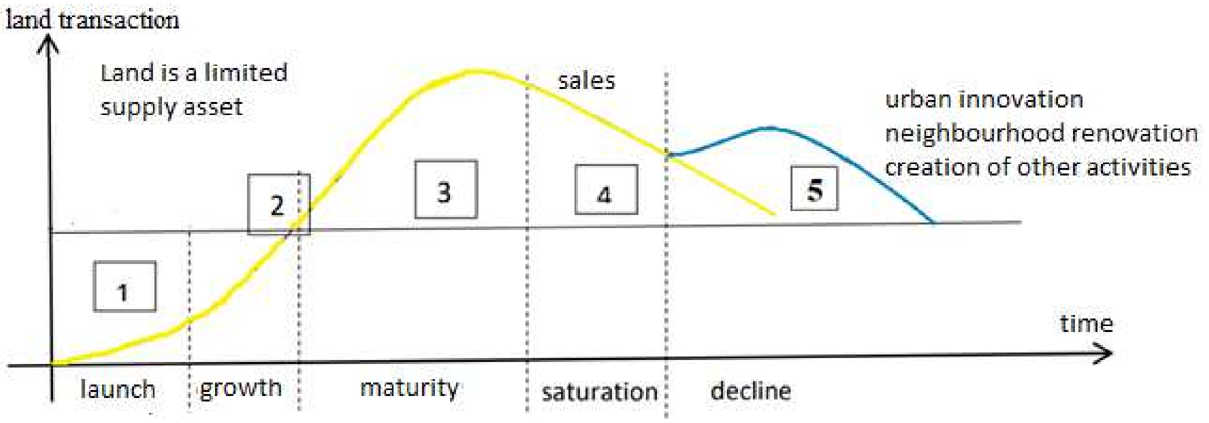

Figure 1 – Land Change Curve

This curve can be considered as a function defined by pieces:
- The first piece concerns phases 1 to 3. This part is modeled by a Gompertz function.
- The second piece concerns the saturation and decline phases, which is modelled by a linear function.

For the By-Pass, for this year 2021, we are in phase 3 of the Gompertz curve. It is these first three phases that we model in this thesis. We cannot see data on the saturation and decline phases, so modeling the linear part is still impossible and is not necessary in this thesis.

4.2- Estimation of the number of land transactions along the By-Pass using the Gauss – Newton algorithm

The function to be estimated is of the form: f(x) = $\alpha.(\frac{t}{\beta})^{(\gamma-1)}.e^{(-\frac{t}{\beta})^{\gamma}}$

The table 1 shows the data for this function of the "Number of land transactions" to be estimated:

Table 1: Data of the function of the "Number of land transactions"

Year	t	Number of land transactions
2003	1	4
2004	2	9
2005	3	22
2006	4	37
2007	5	54
2008	6	81
2009	7	105
2010	8	131
2011	9	162
2012	10	186
2013	11	210
2014	12	229
2015	13	243
2016	14	246
2017	15	249
2018	16	252
2019	17	243
2020	18	217

Here is the Matlab script:

```
clear all ; close all ; clc ;
%%%%%%%%%%%%%%% Gauss Newton Transaction Fonciere%%%%%%%%%%%%%%%
idelta(1,1) = 750 ; idelta(2,1) 20 ; idelta(3,1) = 3 ;

xi = [1    2    3    4    5    6    7    8    9    10    11    12    13    14    15    16    17    18 ] ;
yi = [4    9    22   37   54   81   105  131  162  186   210   229   243   246   249   252   243   217] ;

err = 0.3. *randn(1,numel(yi)) ; it = 0 ;
%%%%%%%%%%%%%%% CALCUL FORMEL DES DERIVEE %%%%%%%%%%%%%%%
syms p1 p2 p3 x
modelLandTransaction = p3. *(x./p1).^(p2-1). *exp(-(x./p1).^p2);

dy_d1 = diff(modelLandTransaction,p1) ; dy_d2 = diff(modelLandTransaction,p2) ;
dy_d3 = diff(modelLandTransaction,p3) ; dy_d11 = subs(dy_d1,x,xi) ;
dy_d22 = subs(dy_d2,x,xi) ; dy_d33 = subs(dy_d3,x,xi);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 itmax = 100 ; tol = 1e-10 ;
while it < itmax

it = it + 1 ;
 dy_da= subs(dy_d11, { 'p1 ' , 'p2 ' , 'p3 '} , {idelta(1)   idelta(2)   idelta(3)});
 dy_db= subs(dy_d22, { 'p1 ' , 'p2 ' , 'p3 '} , {idelta(1)   idelta(2)   idelta(3)});
dy_dc=  subs(dy_d33, { 'p1 ' , 'p2 ' , 'p3 '} , {idelta(1)   idelta(2)   idelta(3)});

    Jacob(1:numel(xi),  1) =  dy_da ;  Jacob(1:numel (xi),  2) =  dy_db ;
    Jacob(1:numel(xi), 3) =  dy_dc ;

 F = yi - subs(modelLandTransaction, { ' x ' , 'p1 ' , 'p2 ' , 'p3 '} , ...
     {xi    idelta(1)    idelta(2)    idelta(3)});

ndelta = inv(Jacob' *Jacob) *(Jacob' *F')  +  idelta ;  idelta = ndelta ;
fx = subs(modelLandTransaction,{ ' x ' , 'p1 ' , 'p2 ' , 'p3 '}, ...
      {xi   idelta(1)   idelta(2)   idelta(3)})  ;

Fn(it)  =  sum(F) ;
rmse(it+1)  =  sqrt(Fn(it).^2)/(length(yi) - length(idelta))  ;

If  rmse(it + 1) - rmse(it)  <  tol
         sol  =  idelta ;
    break
end

%%%%%%%%%%%%%%%%% GRAPHICAL DISPLAY %%%%%%%%%%%%%%%%%%%
n = 500 ; step = (xi(end) - xi(1))/n  ;  xn = xi(1)  : step: xi(end)  ;
 fun  =  subs(modelLandTransaction,  { ' x ' , 'p1 ' , 'p2 ' , 'p3 '} ,  {xn   sol(1)   sol(2)   sol(3)});

figure ( 'color ',[1 1 1]) ; errorbar(xi,yi,err, ' s ' ) ; hold on ;
```

```
plot(xn, fun, ' r ' , 'LineWidth ',2)  ;  xlim([-2   22])  ;  format long
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
h = gca ;
str = { '$${\it p_3}\times \left ( {\frac { x}{{\it p_1}}} \right ) ^{{\it p_2 }-1}\,{ e^{-\left ( {\frac { x}{{\it p_1}}}\right )^{{\it p_2}}}}$$ ' } ;
set (gcf , 'CurrentAxes ',h) ;
text ( 'Interpreter ' , 'latex ' , 'String ',str, 'Position ',[2   0.2], 'FontSize ',14)
```

the result obtained is : **number Land Transaction(t) =** $\mathbf{652.\left(\frac{t}{16{,}71}\right)^{(2{,}07)}.e^{\left(-\frac{t}{16{,}71}\right)^{3{,}07}}}$

5- DISCUSSION - LEVENBERG-MARQUARDT METHOD: A GENERALIZATION OF THE GAUSS–NEWTON METHOD

When the Gauss-Newton method fails, especially when the solution of the sub-problem of the linear system solution, that is to say the Jacobian matrix, is not of complete rank, the Levenberg-Marquardt method is an interesting alternative. In this method we approach the matrix $[J(x^{(k)})]^t . J(x^{(k)})$ by a matrix diagonal $\mu.I_{dentité}$. We then have the following algorithm:

Choose $x^{(0)}$
for k = 0, 1, 2,…. until convergence do
  compute $J(x^{(k)})$ and μ_k
  solve $\{[J(x^{(k)})]^t . J(x^{(k)}) + \mu_k I_d \}. a^{(k)}_{GN} = - [J(x^{(k)})]^t . r(x)$
  update $a^{(k+1)} = a^{(k)} + a^{(k)}_{LM}$
end for

NB : $\mu_k \geq 0$. Thus, for a null μ, we return to the Gauss-Newton algorithm. The parameter μ is normally modified with each iteration. In general, (we admit it in this thesis), the value of μ is of the order of 10^{-2}. Thus, the Levenberg-Marquardt method is used only when in the Gauss Newton method we encounter a Jacobian matrix which is not of complete rank.

6- CONCLUSION

The dynamism of land tenure has a close relationship with the evolution of urbanization. The number of land transactions modelled in this research highlights the close relationship between urban sprawl and urban densification.

An interpretation, of this number of transactions over time, pleads for remarks in terms of public policies. We need to think about structural policies to increase the elasticity of land supply: simplification and acceleration of land transfer procedures in a controlled legal framework.

BIBLIOGRAPHICAL REFERENCES

- [1] A.Nemirovskii and D. Yudin. Problem complexity and methode
- [2] C. Zalinescu. Convex analysis in general vector spaces. World Scientific, 2002.
- [3] K. Kiwiel. A nonderivative version of the gradient sampling algorithm for nonsmooth nonconvex optimization. SIAM Journal on Optimization, 20(4) :1983–1994, 2010

- [4] E. W. Cheney and A. A. Goldstein. Newton’s method for convex programming and Tchebycheff approximation. Numerische Mathematik, 1 :253–268, 1959

ACKNOWLEDGMENTS

The authors want first to give thanks to the Lord, without whom nothing of the smallest details of our lives would have taken place. His grace has provided us with faith, strength, health needed for this research, and to reach the terms of this publication, which is the result of several years of research in the Polytechnic School Antananarivo that we express our deep gratitude and especially its director, Professor Rijalalaina Rakotoson.

We are also thankful to the Editorial Board of the Madarevues And Mada-Hary, as well as their team.
We also need to thank all those who have directly or indirectly contributed to this publication’s realization. Last but not the least we do not forget especially, our family who was always at our side during the development of our research.

The authors.